International Finance
Chapter 6: Foreign Exchange Market Efficiency

Zichao Yang

Zhongnan University of Economics & Law

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Violations of Uncovered Interest Parity

Uncovered Interest Parity

$$1 + i_t = (1 + i_t^*)\frac{E_t(S_{t+1})}{S_t}$$

However, UIP is repeated rejected by real world data. The ensuing challenge is then to understand why UIP fails.

We cover three possible explanations:

- Forward foreign exchange rate contains a risk premium.
- Rational market participants may have an incomplete understanding of the economy and make systematic prediction errors (peso-problem).
- Some market participants are irrational, and they are called “noise traders”.

Deviations from UIP

Let s be the log spot exchange rate, f be the log one-period forward rate, i (i^*) be the one-period nominal interest rate on a domestic (foreign) currency. If uncovered interest parity holds:

$$i_t - i_t^* = E_t(s_{t+1}) - s_t$$

By covered interest parity, we also have:

$$i_t - i_t^* = f_t - s_t$$

Hence, uncovered interest parity equals to:

$$f_t = E_t(s_{t+1})$$

Hansen and Hodrick try to test if UIP holds.

Hansen and Hodrick's Test of UIP

$f_{t,3}$: log 3-month forward exchange rate at time t

s_t: log spot rate at time t

I_t: the information set available to market participants at time t

J_t: the information set available to you, an individual at time t

Hansen and Hodrick want to test the hypothesis:

$$H_0: \quad E(s_{t+3}|I_t) = f_{t,3}$$

However, I_t is not observable, but J_t is a subset of I_t, so what they really test is:

$$H_0': \quad E(s_{t+3}|J_t) = f_{t,3}$$

Hansen and Hodrick's Test of UIP

The regression can be written as:

$$s_t - f_{t-3,3} = z'_{t-3}\underline{\beta} + \epsilon_{t,3}$$

where z_{t-3} is a vector of economic variables in J_{t-3}.

Even though we are working with 3-month forward rates, we will sample the data monthly. Eventually, we wan to do a joint test to see if the slope coefficients $\underline{\beta}$ are zero.

The error term is formed at time $t-3$, $\epsilon_t = s_t - E(s_t|J_{t-3})$, so at $t-3$ we have

$$E(\epsilon_t|J_{t-3}) = E(s_t - E(s_t|J_{t-3})) = 0$$

which means the error term is unpredictable at time $t-3$ when it is formed. Then how about $E(\epsilon_t|J_{t-1})$?

Hansen and Hodrick's Test of UIP

At time $t-2$ and $t-1$ you will get new information, so:

$$E(\epsilon_t|J_{t-1}) = E(s_t|J_{t-1}) - E[E(s_t|J_{t-3})|j_{t-1}] \neq 0$$

Hence, the first auto-covariance of the error term

$$E(\epsilon_t \epsilon_{t-1}) = E(\epsilon_{t-1} E(\epsilon_t|J_{t-1}))$$

need not to be zero.

Actually we can only say that $E(\epsilon_t \epsilon_{t-k}) = 0$ for $k \geq 3$.

If you work with a $k-period$ forward rate, you must be prepared for the error term to follow an MA(k-1) process.

In their regressions, Hense and Hodrick calculate the asymptotic covariance matrix assuming that the regression error follows a moving average process.

Hansen and Hodrick's Test of UIP

Table 6.1: Hansen-Hodrick tests of UIP

	US-BP	US-JY	US-DM	DM-BP	DM-JY	BP-JY
Wald(NW[6])	16.23	400.47	5.701	66.77	46.35	294.31
p-value	0.001	0.000	0.127	0.000	0.000	0.000
Wald(HH[2])	16.44	324.85	4.299	57.81	32.73	300.24
p-value	0.001	0.000	0.231	0.000	0.000	0.000

Notes: Regression $s_t - f_{t-3,3} = \underline{z}'_{t-3}\underline{\beta} + \epsilon_{t,3}$ estimated on monthly observations from 1973,3 to 1999,12. Wald is the Wald statistic for the test that $\underline{\beta} = 0$. Asymptotic covariance matrix estimated by Newey-West with 6 lags (NW[6]) and by Hansen–Hodrick with 2 lags (HH[2]).

In these data, UIP is rejected for every currency except for the dollar-deutsche mark rate.

The Advantage of Using Overlapping Observations

You can always avoid inducing the serial correlation into the regression error by only using non-overlapping observations. But, then, you will have less observations to work with.

Overlapping monthly observations increases the nominal sample size by a factor of 3 but the effective increase in sample size may be less than this if the additional obs. are highly dependent.

Table 6.2: Monte Carlo Distribution of OLS Slope Coefficients and T-ratios using Overlapping and Nonoverlapping Observations.

T	Overlapping Observations		percentiles 2.5	50	97.5	Relative Range
50	yes	slope	0.778	0.999	1.207	0.471
		t_{NW}	(-2.738)	(-0.010)	(2.716)	1.207
		t_{HH}	[-2.998]	[-0.010]	[3.248]	1.383
16	no	slope	0.543	0.998	1.453	
		t_{OLS}	((-2.228))	((-0.008))	((2.290))	
100	yes	slope	0.866	0.998	1.126	0.474
		t_{NW}	(-2.286)	(-0.025)	(2.251)	1.098
		t_{HH}	[-2.486]	[-0.020]	[2.403]	1.183
33	no	slope	0.726	0.996	1.274	
		t_{OLS}	((-2.105))	((-0.024))	((2.026))	
300	yes	slope	0.929	1.001	1.074	0.509
		t_{NW}	(-2.071)	(0.021)	(2.177)	1.041
		t_{HH}	[-2.075]	[-0.016]	[2.065]	1.014
100	no	slope	0.858	1.003	1.143	
		t_{OLS}	((-2.030))	((0.032))	((2.052))	

Fama Decomposition Regressions

Define the expected excess nominal forward foreign exchange payoff to be:

$$p_t \equiv f_t - E_t[s_{t+1}]$$

From the Hansen-Hodrick regressions we already know that p_t is non zero. Adding and subtracting s_t from both sides can get:

$$f_t - s_t = E_t(s_{t+1} - s_t) + p_t$$

Fama deduces some properties of p_t using the following two regressions:

(1) consider the regression of the ex post forward profit $f_t - s_{t+1}$ on the current period forward premium $f_t - s_t$

$$f_t - s_{t+1} = \alpha_1 + \beta_1(f_t - s_t) + \epsilon_{1,t+1}$$

(2) consider the regression of the one-period ahead depreciation $s_{t+1} - s_t$ on the current period forward premium

$$s_{t+1} - s_t = \alpha_2 + \beta_2(f_t - s_t) + \epsilon_{2,t+1}$$

Fama Decomposition Regressions

Table 6.3: Estimates of Regression Equations (6.3) and (6.4)

	US-BP	US-JY	US-DM	DM-BP	DM-JY	BP-JY
$\hat{\beta}_2$	-3.481	-4.246	-0.796	-1.645	-2.731	-4.295
$t(\beta_2 = 0)$	(-2.413)	(-3.635)	(-0.542)	(-1.326)	(-1.797)	(-2.626)
$t(\beta_2 = 1)$	(-3.107)	(-4.491)	(-1.222)	(-2.132)	(-2.455)	(-3.237)
$\hat{\beta}_1$	4.481	5.246	1.796	2.645	3.731	5.295

Notes: Nonoverlapping quarterly observations from 1976.1 to 1999.4. $t(\beta_2 = 0)$ ($t(\beta_2 = 1)$ is the t-statistic to test $\beta_2 = 0$ ($\beta_2 = 1$).

The $\bar{\beta}_2$ is significantly less than 1, so uncovered interest parity is rejected.

Furthermore, $\bar{\beta}_2$ is actually negative, and this phenomenon is called **Forward Premium Puzzle**. The forward premium evidently predicts the future depreciation but with the "wrong" sign from the UIP perspective.

Fama Decomposition Regressions

This anomaly is driven by the dynamics in p_t. We can writ β_1 and β_2 as (How?):

$$\beta_1 = \frac{Var(p_t) + Cov[p_t, E_t[\Delta s_{t+1}]]}{Var(p_t) + Var[E_t(\Delta s_{t+1})] + 2Cov[p_t, E_t(\Delta s_{t+1})]}$$
$$\beta_2 = \frac{Var[E_t(\Delta s_{t+1})] + Cov[p_t, E_t[\Delta s_{t+1}]]}{Var(p_t) + Var[E_t(\Delta s_{t+1})] + 2Cov[p_t, E_t(\Delta s_{t+1})]}$$

Based on the regression results, we can get:

$$\left.\begin{matrix} \beta_1 > 0 \\ \beta_2 < 0 \end{matrix}\right\} \Rightarrow \begin{cases} Cov[p_t, E_t[\Delta s_{t+1}]] < 0 \\ Var(p_t) > |Cov[p_t, E_t[\Delta s_{t+1}]]| > Var[E_t(\Delta s_{t+1})] \end{cases}$$

Estimating p_t

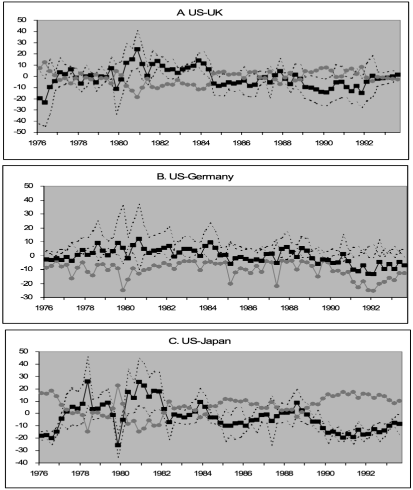

Figure 6.1: Time series point estimates of p_t (boxes) with 2-standard error bands and point estimates of $E_t(\Delta s_{t+1})$ (circles).

Rational Risk Premia

In this model, we assume the representative agent's intertemporal marginal rate of substitution between t and $t+1$ can be written as

$$\mu_{t+1} = \frac{\beta u'(C_{t+1})}{u'(C_t)}$$

where $u'(C_t)$ is the representative agent's marginal utility evaluated at equilibrium consumption.

Intertemporal Marginal Rate of Substitution & Good State:

When the agent experiences the good state, consumption growth is high and the intertemporal marginal rate of substitution is low.

Rational Risk Premia

If the agent is behaving optimally, the expected marginal utility from the real payoff from buying the foreign currency forward is:

$$E_t[u'(C_{t+1})\frac{F_t - S_{t+1}}{P_{t+1}}] = 0$$

We can rewrite this equation by multiplying both sides by β and dividing by $u'(C_t)$ to get:

$$E_t[\mu_{t+1}\frac{F_t - S_{t+1}}{P_{t+1}}] = 0$$

Now there is no arbitrage opportunity in the market. This equation will help us understand the risk-premia stemmed from the demand for forward foreign exchange, and explain why UIP does not hold.

Covariance Decomposition

Suppose we have two random variables X_{t+1} and Y_{t+1}, then the covariance is

$$COV_t(X_{t+1}, Y_{t+1}) = E_t(X_{t+1}Y_{t+1}) - E_t(X_{t+1})E_t(Y_{t+1})$$

If we further assume $E_t(X_{t+1}Y_{t+1})$, then we can get:

$$E_t(Y_{t+1}) = \frac{-Cov_t(X_{t+1}, Y_{t+1})}{E_t(X_{t+1})}$$

Now if we set $Y_{t+1} = \frac{F_t - S_{t+1}}{P_{t+1}}$ and $X_{t+1} = \mu_{t+1}$, we can get:

$$E_t[\frac{F_t - S_{t+1}}{P_{t+1}}] = \frac{-Cov_t[\frac{F_t - S_{t+1}}{P_t}, \mu_{t+1}]}{E_t\mu_{t+1}}$$

The Real Risk Premium

$$E_t[\frac{F_t - S_{t+1}}{P_{t+1}}] = \frac{-Cov_t[\frac{F_t - S_{t+1}}{P_t}, \mu_{t+1}]}{E_t \mu_{t+1}}$$

This equation shows us that

- The forward rate is in general not the rationally expected future spot in this model.
- The expected forward contract payoff is proportional to the conditional covariance between the payoff and the intertemporal marginal rte of substitution.

How do we make sense of this equation?

The Real Risk Premium

Suppose that $E_t[\frac{F_t - S_{t+1}}{P_{t+1}}] < 0$, what does this mean?

This means if you buy the foreign currency (euros) forward and resell them in the spot market at period $t+1$, you will make a profit. It seems like that the euro is a risky currency and the market pays a premium to those who are willing to hold euro-denominated assets.

But...why is the euro a risky asset?

When $E_t[\frac{F_t - S_{t+1}}{P_{t+1}}] < 0$, $Cov_t[\frac{F_t - S_{t+1}}{P_t}, \mu_{t+1}]$ should be positive, then a smaller negative $E_t[\frac{F_t - S_{t+1}}{P_{t+1}}]$ (i.e. larger risk premium) should be associated with a lower μ_{t+1}, which indicates a high consumption growth rate. So holding the euro forward pays off well in good states, but pays off poorly in bad states. This is not an ideal asset for risk-averse investors. Risk-averse investors hope their assets pay off well in bad states.

The Real Risk Premium

Hence, the euro is a risky asset and the UID does not hold because the market needs to pay the holders a risk premium.

Q: if the euro is risky, then how about the dollar?

If $E_t[\frac{F_t - S_{t+1}}{P_{t+1}}] < 0$ and you buy the dollar forward, you expect to realize a loss.

Risk-averse investors are willing to hold dollar-denominated assets because these assets provide consumption insurance by providing high payoff in bad (low growth) consumption states. The expected negative payoff can be viewed as an insurance premium. So now the dollar is a safe asset.

Risk-neutral Forward Exchange

If individuals are risk neutral, the intertemporal marginal rate of substitution μ_{t+1} is constant. (Why?)

Then the equation $E_t[\frac{F_t - S_{t+1}}{P_{t+1}}] = \frac{-Cov_t[\frac{F_t - S_{t+1}}{P_t}, \mu_{t+1}]}{E_t \mu_{t+1}}$ can be rewritten as:

$$E_t(\frac{F_t}{P_{t+1}}) = E_t(\frac{S_{t+1}}{P_{t+1}})$$

Even under risk-neutrality, the forward rate is not the rationally expected future spot rate (UIP does not hold) because you need to divide them by the stochastic future price level.

Nominal Risk Premium

Multiply $E_t[\mu_{t+1}\frac{F_t-S_{t+1}}{P_{t+1}}]=0$ by P_t and divide through by S_t to get:

$$E_t[(\mu_{t+1}\frac{P_t}{P_{t+1}})(\frac{F_t-S_{t+1}}{S_t})]=E_t[\mu^m_{t+1}(\frac{F_t-S_{t+1}}{S_t})]=0$$

where $\mu^m_{t+1}=\mu_{t+1}\frac{P_t}{P_{t+1}}$, and we call μ^m_{t+1} the intertemporal marginal rate of substitution of money.

Based on the same logic in Real Risk premium section, here we can get:

$$E_t[\frac{F_t-S_{t+1}}{S_t}]=(1+i_t)Cov_t[\mu^m_{t+1},\frac{S_{t+1}}{S_t}]$$

Nominal Risk Premium

$$E_t[\frac{F_t - S_{t+1}}{S_t}] = (1 + i_t)Cov_t[\mu^m_{t+1}, \frac{S_{t+1}}{S_t}]$$

Now, if $E_t[\frac{F_t - S_{t+1}}{S_t}] < 0$, the covariance will be negative. In the bad state, μ^m_{t+1} is high because consumption growth is low. This is associated with a weakening of the euro (low value of $\frac{S_{t+1}}{S_t}$). Hence, the euro is risky because its value is positively correlated with consumption.

The foreign currency buys fewer foreign goods in the bad state and is therefore a bad hedge against low consumption states.

Pitfalls of the Risk Premium Explanation

In order to explain the real world data, we need people to be very risk averse otherwise risk premium itself is not sufficient.

Some Survey Results

In some surveys, economists analyze forecasts on 10 USD bilateral rates and 8 deutsche mark bilateral rates from 1986-01 to 1990-12. The survey respondents were asked to provide forecasts at horizons of 3, 6, and 12 months into the future. We will focus on two regressions.

(1) Let $\bar{s}^e_{t+1}$ be the median of the survey forecast of the log spot exchange rate s_{t+1} reported at date t. We regress the survey forecast error on the forward premium.

$$\Delta\bar{s}^e_{t+1} - \Delta s_{t+1} = \alpha_1 + \beta_1(f_t - s_t) + \epsilon_{1,t+1}$$

If survey respondents have rational expectations, the survey forecast error realized at date $t+1$ will be uncorrelated with any publicly available info at time t and the coefficient of β_1 should be zero.

Some Survey Results

(2) this regression measures the weight that market participants attach to the forward premium in their forecasts of the future depreciation.

$$\Delta \bar{s}^e_{t+1} = \alpha_2 + \beta_2 (f_t - s_t) + \epsilon_{2,t+1}$$

Here we want to check if $\beta_2 = 1$. If a risk premium exists, it should be impounded in the regression error and cause β_2 to deviate from 1 through the omitted variables bias.

Some Survey Results

Table 6.4: Empirical Estimates from Studies of Survey Forecasts

	Data Set					
	Economist	MMS	AMEX	CFD	BIC–USD	BIC–DEM
Horizon: 3-months						
β_1	2.513	6.073	—	—	5.971	1.930
$t(\beta_1 = 1)$	1.945	2.596	—	—	1.921	-0.452
$t(\beta_2 = 1)$	1.304	-0.182	—	0.423	1.930	0.959
t-test	1.188	-2.753	—	-2.842	5.226	-1.452
Horizon: 6-months						
β_1	2.986	—	3.635	—	5.347	1.841
$t(\beta_1 = 1)$	1.870	—	2.705	—	2.327	-0.422
β_2	1.033	—	1.216	—	1.222	0.812
$t(\beta_2 = 1)$	0.192	—	1.038	—	1.461	-4.325
Horizon: 12-months						
β_1	0.517	—	3.108	—	5.601	1.706
$t(\beta_1 = 1)$	0.421	—	2.400	—	3.416	0.832
β_2	0.929	—	0.877	1.055	1.046	0.502
$t(\beta_2 = 1)$	-0.476	—	-0.446	0.297	0.532	-6.594

Some Survey Results

Two main points can be drawn from the table.

- The coefficient of β_1 is significantly different from zero which provides evidence against the rationality of the survey expectations.
- Estimates of β_2 are generally insignificantly different from 1, which suggests that survey respondents do not believe that there is a risk premium in the forward foreign exchange rate.

A Simple Peso Example

In previous settings, we always assume that agents have prefect knowledge about the economy. In “peso problem” we will loose this assumption and assume that agents may have imperfect knowledge about some aspects of the underlying economic environment.

The “peso problem” was originally studied by Krasker who observed a persistent interest difference in favor of Mexico even though the nominal exchange rate was fixed by the central bank. By covered interest arbitrage, we get:

$$i_t - i_t^* = f_t - s_t < 0$$

If the fixed exchange rate s_t is maintained at $t+1$, then we have a realization of $f_t < s_t = s_{t+1}$, there is a persistent forward rate forecast error in the system.

A Simple Peso Example

How can we explain this persistent forecast error?

Suppose there is a probability p that the Mexico central bank will abandon the peg and devalue the peso to $s_1 < s_0$ and a probability $1 - p$ that the s_0 peg will be maintained. The process governing the exchange rate is

$$s_{t+1} = \begin{cases} s_1 & \text{with probability } p \\ s_0 & \text{with probability } 1 - p \end{cases}$$

Then the 1-period ahead rationally expected future spot rate should be

$$E_t(s_{t=1}) = ps_1 + (1 - p)s_0$$

So we will observed a persistent and rational forecast error:

$$E_t(s_{t+1}) - s_0 = p(s_1 - s_0) < 0$$

Noise Traders Theory

The Noise Trader Model consider the possibility that some market participants are not fully rational. The model adapts the overlapping-generations noise trader model of De Long et.al. to study the pricing of foreign currencies in an environment.

- Heterogeneous beliefs across agents: rational (fundamental) and noise traders
- noise traders are unable to distinguish between pseudo-signals and news
- short-horizon rational investors bear the risk that they may be required to liquidate their positions at a time

The Model

Two countries: domestic country (dollar) and foreign country (euro).

The price level in each country is fixed at unity in local currency, $P = P^* = 1$. Individuals therefore evaluate wealth in local currency units.

λ_t is the dollar value of the portfolio position taken. $\lambda_t > 0$ means a long position in euro.

A long position in the euro means:

- invest λ_t dollar to buy λ_t / S_t euros, opportunity cost at $t+1$ is $\lambda_t R_t$
- in $t+1$, the euro payoff is $R_t^*(\lambda_t / S_t)$, which equals $(S_{t+1}/S_t) R_t^* \lambda_t$ in dollars
- net payoff is $[(S_{t+1}/S_t) R_t^* - R_t]\lambda_t$

The Model

We use the approximations $(S_{t+1}/S_t) \simeq (1 + \Delta s_{t+1})$ and $(R_t/R_t^*) = (F_t/S_t) \simeq 1 + x_t$ to express the domestic agent's net dollar payoff as:

$$[\Delta s_{t+1} - x_t]R_t^* \lambda_t$$

The foreign agent's portfolio position is denoted by λ_{*t}. (Note: λ_{*t} is also denominated in dollars.)

$\lambda_{*t} > 0$ indicates a long euro position, and the next period's net euro payoff is $(R_t^*/S_t - R_t/S_{t+1})\lambda_{*t}$.

$\lambda_{*t} < 0$ indicates a long dollar position, and the next period's net euro payoff is $-(R_t/S_{t+1} - R_t^*/S_t)\lambda_{*t}$.

The Model

Using the approximation $\frac{(R_t/R_t^*)}{S_{t+1}/S_t} \simeq 1 + x_t - \Delta s_{t+1}$, the foreign agent's net euro payoff is:

$$[\Delta s_{t+1} - x_t] R_t^* \frac{\lambda_{*t}}{S_t}$$

Also, the foreign exchange market clears when net dollar sales of the current young equals net dollar purchases of the current old:

$$\lambda_t + \lambda_{*t} = \frac{S_t}{S_{t-1}} R_{t-1}^* \lambda_{t-1} + R_{t-1} \lambda_{*t-1}$$

Fundamental and Noise Traders

A fraction μ of domestic and foreign traders are fundamentalists who have rational expectations. The remaining fraction $1-\mu$ are noise traders whose beliefs are distorted.

The speculative positions of home fundamentalists and noise traders are λ_t^f and λ_t^n.

The speculative positions of foreign fundamentalists and noise traders are λ_{*t}^f and λ_{*t}^n.

The total portfolio position of domestic agents is $\lambda_t = \mu\lambda_t^f + (1-\mu)\lambda_t^n$.

The total portfolio position of foreign agents is $\lambda_{*t} = \mu\lambda_{*t}^f + (1-\mu)\lambda_{*t}^n$.

Fundamental and Noise Traders

The utility function that every agent wants to maximize is a constant absolute risk aversion utility function:

$$E_t(-e^{-\gamma W_{t+1}})$$

where W_{t+1} is the agent's wealth in $t+1$.

This maximization problem equals to the following problem:

$$\max \quad E_t(W_{t+1}) - \frac{\gamma}{2} V_t(W_{t+1})$$

where $V_t(W_{t+1})$ is the variance of W_{t+1}.

Let us begin with a economy that contains only fundamentalists.

A fundamentalists ($\mu = 1$) Economy

The wealth of the fundamentalist domestic agent is the portfolio payoff plus c dollars of exogenous "labor" income.

x_t is a AR(1) process:

$$x_t = \rho x_{t-1} + v_t$$

with $\rho \in (0, 1)$ and $v_t \overset{iid}{\sim} (0, \sigma_v^2)$.

Without loss of generality, we let the uncertainty in the economy is driven by R_t alone and fix $R^* = 1$.

The domestic agent evaluate the conditional mean and variance of next period wealth:

$$E_t(W_{t+1}^f) = [E_t(\Delta s_{t+1}) - x_t]\lambda_t^f + c$$
$$V_t(W_{t+1}^f) = \sigma_s^2(\lambda_t^f)^2$$

A fundamentalists ($\mu = 1$) Economy

The maximization problem is:

$$\max \quad [E_t(\Delta s_{t+1}) - x_t]\lambda_t^f + c - \frac{\gamma}{2}\sigma_s^2(\lambda_t^f)^2$$

And the solution is:

$$\lambda_t^f = \frac{[E_t(\Delta s_{t+1}) - x_t]}{\gamma \sigma_s^2}$$

Implications:

- $E_t(\Delta s_{t+1}) \uparrow$, USD will depreciate, agents will borrow more ($\lambda_t \uparrow$) USD, take a long position in EUR
- if $x_t \downarrow$, then $\frac{R_t}{R_t^*} \downarrow$, USD will be less attractive, so $\lambda_t \uparrow$.
- people are risk averse, so if $\gamma\sigma_s^2 \uparrow$, people will allocate less wealth on foreign assets, so $\lambda_t \downarrow$.

A fundamentalists ($\mu = 1$) Economy

Following the same logic, we can get the optimal position for the foreign agent: $\lambda^f_{*t} = S_t \lambda^f_t$.

Based on the market clearing condition deducted in previous slides we can get:

$$E_t \Delta s_{t+1} - x_t = \Gamma_t (E_{t-1} \Delta s_t - x_{t-1})$$

where $\Gamma_t \equiv \frac{[(S_t/S_{t-1}) + S_{t-1} R_{t-1}]}{1+S_t}$

One solution for the rate of depreciation is:

$$\Delta s_t = \frac{1}{\rho} x_t = x_{t-1} + \frac{1}{\rho} v_t$$

So $E_t(\Delta s_{t+1}) = x_t$, there is no forward premium bias if agents are fundamentalists.

A Noise Trader ($\mu < 1$ Economy)

Noise traders believe that assets returns are influenced by other factors ($\{n_t\}$).

When $n - t > 0$ noise traders believe the dollar will be weaker in the future than what is justified by the fundamentals.

$$n_t = kx_t + u_t$$

where $k > 0$, and $u_t \overset{iid}{\sim} (0, \sigma_u^2)$.

The distortion in noise trader beliefs occurs only in evaluating first moments of returns. Their evaluation of second moments coincide with those of fundamentalists.

$$E_t(W_{t+1}^n) = [E_t(\Delta s_{t+1}) - x_t]\lambda_t^n + n_t\lambda_t^n + c$$
$$V_t(W_{t+1}^n) = \sigma_s^2(\lambda_t^n)^2$$

A Noise Trader ($\mu < 1$ Economy)

The domestic noise trader's problem is to maximize:

$$\lambda_t^n(E_t\Delta s_{t+1} - x_t + n_t) - \gamma\sigma_s^2(\lambda_t^n)^2$$

and the solution is:

$$\lambda_t^n = \lambda_t^f + \frac{n_t}{\gamma\sigma_s^2}$$

For foreign noise trader, the optimal position is $\lambda_{*t}^n = S_t\lambda_t^n$.

The market clearing condition is:

$$(E_t\Delta s_{t+1} - x_t) + (1-\mu)n_t = \Gamma_t[E_{t-1}\Delta s_t - x_{x-t} + (1-\mu)n_{t-1}]$$

and the solution is:

$$\Delta s_t = \frac{1}{\rho}x_t - \frac{1-\mu}{\rho}n_t - (1-\mu)u_{t-1}$$

Properties of the Solutions

When noise traders are excessively pessimistic and take short position in the dollar, fundamentalists take the offsetting long position. The market has to be cleared.

Foreign exchange risk, excess currency movements and trading volume are induced entirely by noise traders.

Neither types of trader is guaranteed to earn profits or losses.